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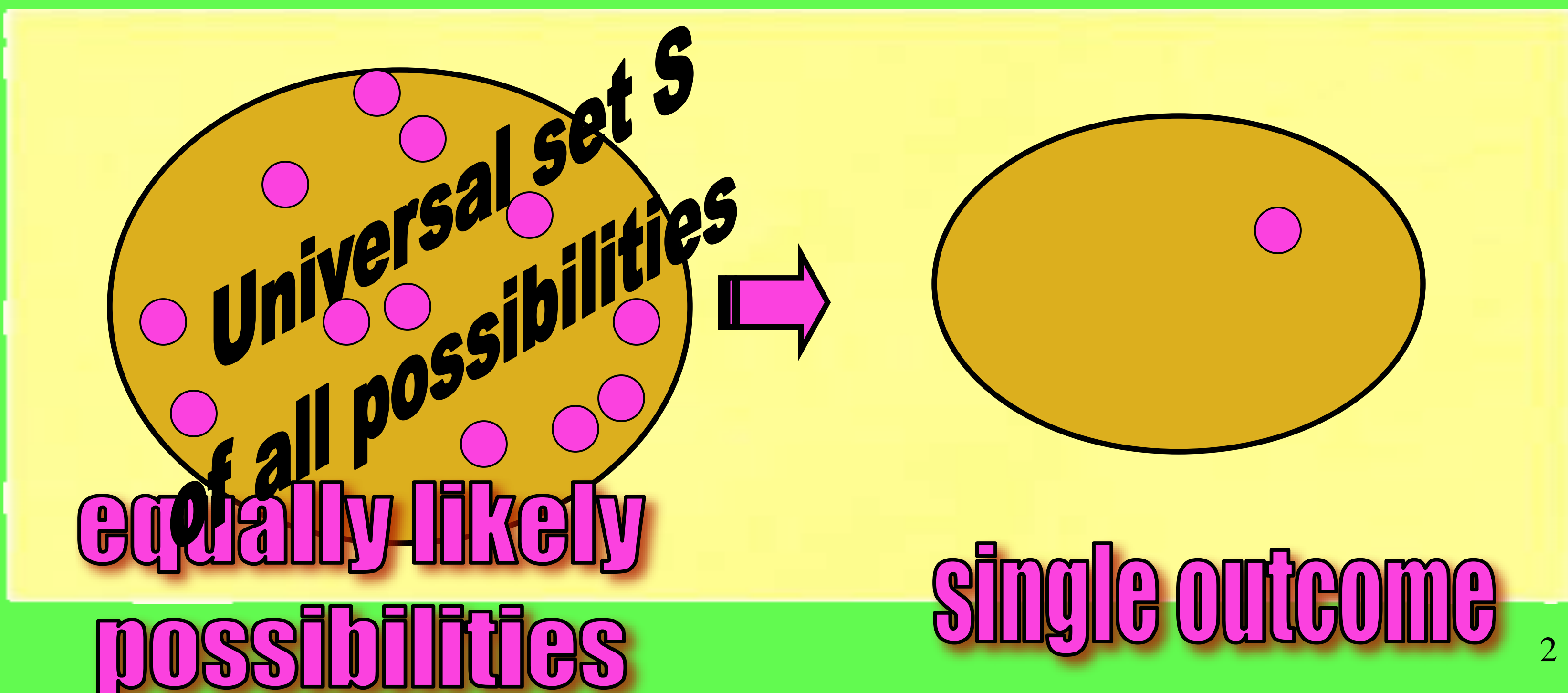
Week 3-16-09

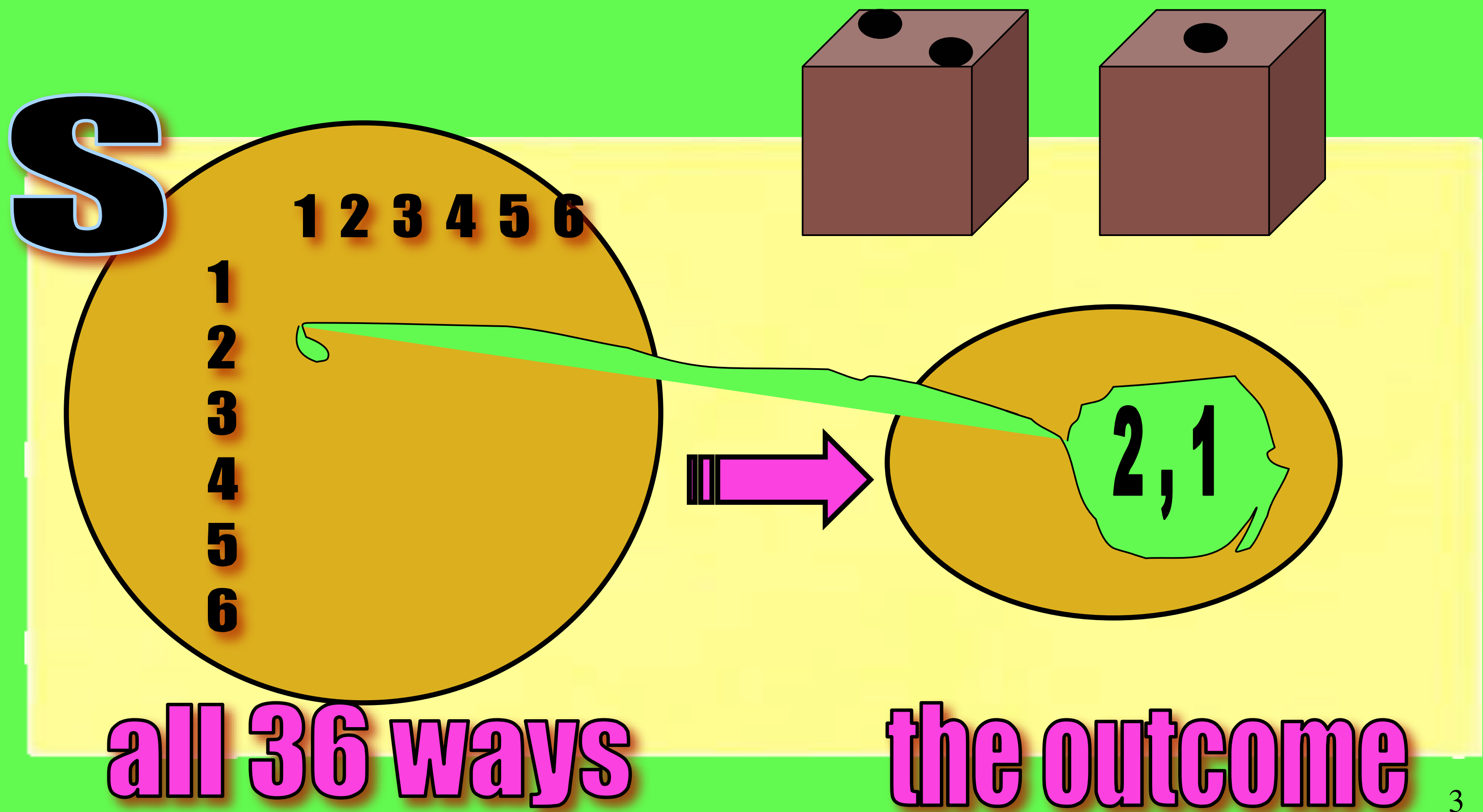
CLASSICAL PROBABILITY

In the classical model all outcomes are regarded as being equally likely.



TOSS OF TWO DICE

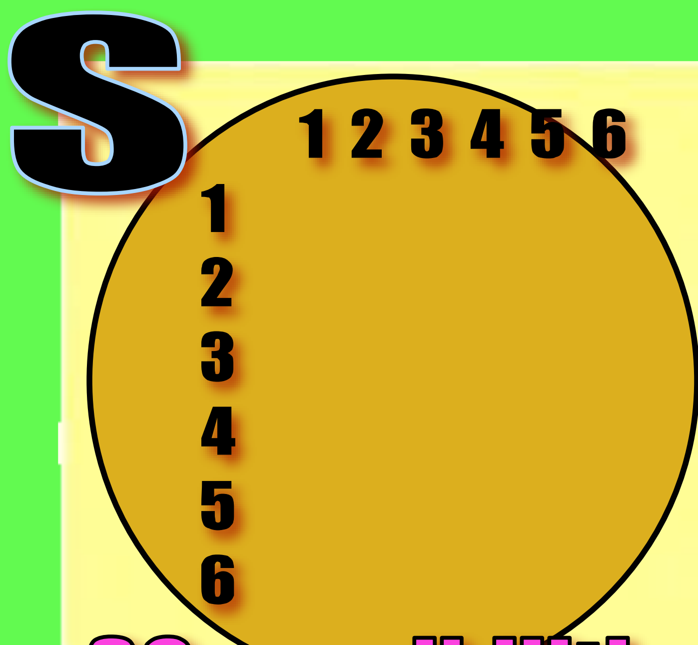
List all 36 ways the dice may fall.



CALCULATING CLASSICAL PROBABILITIES

P(event) = n(event) / n(S)

an event is just a **subset** of the sample space S



36 possibilities for two dice

classical

P(total is 7)

= the ratio of the number of favoring cases to the total possible

CALCULATING CLASSICAL PROBABILITIES

$$P(\text{event}) = n(\text{event}) \,/\, n(S)$$

an event is just a **subset** of the sample space

take the **ratio of favorable outcomes** to total outcomes

	1	2	3	4	5	6
1						*
2					*	
3				*		
4			*			
5		*				
6	*					

check off all cases favorable to the total of 7

P(total is 7) = 6 / 36 = 1/6

CALCULATING CLASSICAL PROBABILITIES

P(event) = n(event) / n(S)

an event is just a **subset** of the sample space

take the **ratio of favorable outcomes** to total outcomes

	1	2	3	4	5	6
1			*			
2		*				
3	*					
4						
5						
6						

check off all cases favorable to the total of 4

P(total is 4)

= 3 / 36 = 1/12

DISTRIBUTION FOR THE TOTAL OF TWO DICE

1 2 3 4 5 6

1
2
3
4
5
6

check off all cases favorable to each of totals 2, 3, ... , 11, 12

total	favorable cases
2	1
3	2
4	3
...	
7	6
8	5
...	
12	1

e.g. P(10) = 3/36 = 1/12 from 6, 4 or 5, 5 or 4, 6.

THE GENETIC LOTTERY

Mom

A a

A

Dad

a

check off all cases favorable to each of types aa, aA (same as Aa), AA

type	favorable
aa	1
aA	2
AA	1

P[aa] = 1/4

P[aA] = 2/4 = 1/2

P[AA] = 1/4

LET'S MAKE A DEAL

A prize is behind one of 3 doors. Whatever door you guess the host will reveal another door behind which there is no prize and ask if you wish to switch to the remaining door (not your original pick and not the opened one).

If your original pick is at random then your chance of winning if you always switch is 2/3 since you then only lose if you originally choose the prize door.

If you never switch your win rate is 1/3.

CLASSICAL PROBABILITIES MAY SURPRISE

{$1, $1, $5}

Jack draws a bill first

Jill draws second

from the two bills then remaining

what is

P(Jill $5) ?

what possibilities ?

many people are puzzled by the answer !

GOLD STANDARD: USE AN AGREED UPON MODEL

{\$1, \$1, \$5}

{ a, b, c }

Jack first, Jill second

draw **BILLS** not dollar amounts

		Jill		
		a	b	c
	a	elim		
Jack	b		elim	
	c			elim

all 6 possibilities

calculate

P(Jill \$5)

GOLD STANDARD ANSWER

{\$1a, \$1b, \$5c}

Jack draws a bill first

Jill draws second

from the **two** bills then remaining

		Jill		
		a	b	c
	a	elim		*
Jack	b		elim	*
	c			elim

all 6 possibilities

P(Jill \$5)

= 2 / 6 = 1 / 3

same as Jack

DOES NOT JACK'S DRAW INFLUENCE THE RESULT ?

JILL DRAWS FROM ONLY 2, SO HOW CAN IT BE 1/3 ?

{$1, $1, $5}Jack first, Jill second

from the **two** bills then remaining

Jill $5 = Jack $1 "and" Jill $5

this gets Jack's draw into the picture

MULTIPLICATION RULE HELPS !

{\$1, \$1, \$5} Jack first, Jill second

from the **two** bills then remaining

P(Jill \$5) = P(Jack 1 and Jill 5) =

P(Jack 1) P(Jill 5 "if" Jack 1) =

(2 / 3) (1/ 2)=1/ 3

RATIONALE IN TERMS OF COUNTS

{\$1, \$1, \$5}Jack first, Jill second

from the **two** bills then remaining

1. P(Jack 1) = n(Jack 1) / n(S)
2. P(Jill 5 "if" Jack 1) =
n(Jack 1 and Jill 5) / n(Jack 1).
3. The product of (1) with (2) gives P(Jill 5)

CONDITIONAL PROBABILITY

{$1, $1, $5}Jack first, Jill second

from the **two** bills then remaining

P(Jill 5 given that Jack 1)

IS THE CONDITIONAL PROBABILITY

FOR JILL 5 GIVEN THAT JACK 1

IT IS WRITTEN P(JILL 5 | JACK 1) = 1 / 2

MULTIPLICATION APPLIED TO DRAWING BALLS

draws without replacement
from {R, R, R, B, B, G}

calculate P(B1 R2)
i.e. first ball drawn is black
and second ball drawn is red

MULTIPLICATION RULE WHEN DRAWING BALLS

draws without replacement
from {R, R, R, B, B, G}

1. P(B1) = 2 / 6
2. P(R2 given that B1) = 3 / 5 "write P(R2 | B1)"
3. P(B1 R2) = (2 / 6) (3 / 5) = 1/ 5 (multiplication rule)

MULTIPLICATION FOR MORE THAN TWO

draws without replacement

from {R, R, R, B, B, G}

a. P(B1 G2 B3) = (2/6) (1/5) (1/4) ⇐

same

b. P(G1 B2 B3] = (1/6) (2/5) (1/4) ⇐

c. P(G1 B2 G3) = (1/6) (2/5) (0/4) = 0

BIRTHDAY PROBLEM

Suppose that each birth is independently placed into one of 365 days. The chance that all of a given number n of birthdays will differ is

364/365	**363/365 …**	**(366-n)/365**
2nd misses first	**3rd misses 1st and 2nd**	**etc.**

This is around e^(- n (n-1)/730) ~ 1/2 for n = 23. That is, around 50% of the time there would be no shared birthdays among 23 persons. By complements, there is around 1/2 probability of at least one instance of same birthdays among 23 persons, and even greater in the real world where some days have more births.

LAW OF TOTAL PROBABILITY

reasoning through different causes

draws without replacement from {R, R, R, B, B, G}.

CALCULATE P(R2)

"Jack & Jill" suggests it is 1/2 the same as for draw one

ORDER OF THE DEAL DOES NOT MATTER

In deals without replacement, order of the deal does not matter

e.g. no need to fight over where to sit at cards
(vis-a-vis the cards dealt, skill levels of players may matter)

without replacement draws from { R, R, R, B, B, G }

P(R2) should be the same as

P(R1) = 3 / 6 = 1 / 2

GETTING P(R2) FROM TOTAL PROBABILITY

without replacement draws from {R,R,R,B,B,G}

$$P(\ R1\ \ R2\ ) = (3/6)(2/5)$$

$$P(B1\ \ R2) = (2/6)(3/5)$$

$$P(G1\ \ R2) = (1/6)(3/5)$$

$$P(R2) = \text{sum of above}$$

P(R2) = (3/6)(2/5) + (2/5)(3/5) + (1/6)(3/5)
= 6/30 + 6/30 + 3/30 = 15/30 = 1/2
same as P(R1).

R2 must come by way of exactly one of R1, B1, G1

INDEPENDENT EVENTS

When the occurrence of one thing does not change the probabilities for the other.

events A, B are INDEPENDENT if and only if $P(B \mid A) = P(B)$

DRAWS WITH REPLACEMENT ARE INDEPENDENT

WITH-replacement samples are statistically independent.

P(R1 B2) = P(R1) P(B2 | R1)

P(R1) = 3/6

P(B2 | R1) = 2/6 (with repl).

So P(R1 B2) = (3/6) (2/6) = 1/6.

draws WITH replacement from {R, R, R, B, B, G}

Left handed, Left whorled

Klar proposes the following: If you are type aa for a hypothesized gene controlling handedness then you have 50% chance of being born non-right handed and **independently** of this you have 50% chance of being non-right whorled (hair). If you are not type aa you are born right handed and right whorled.

HANDEDNESS

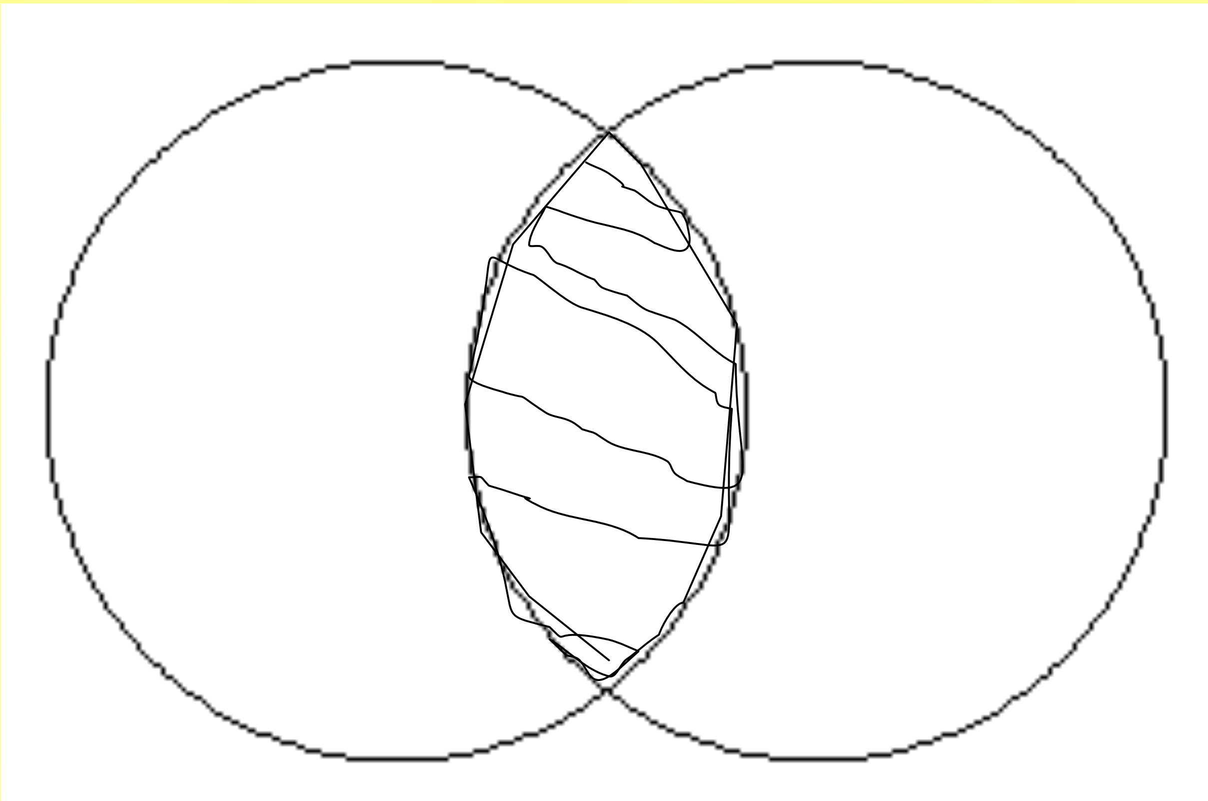
hand \ hair	L	R
L		
R		

check off all cases favorable to each of types LL, LR, RL, RR (hand and hair)

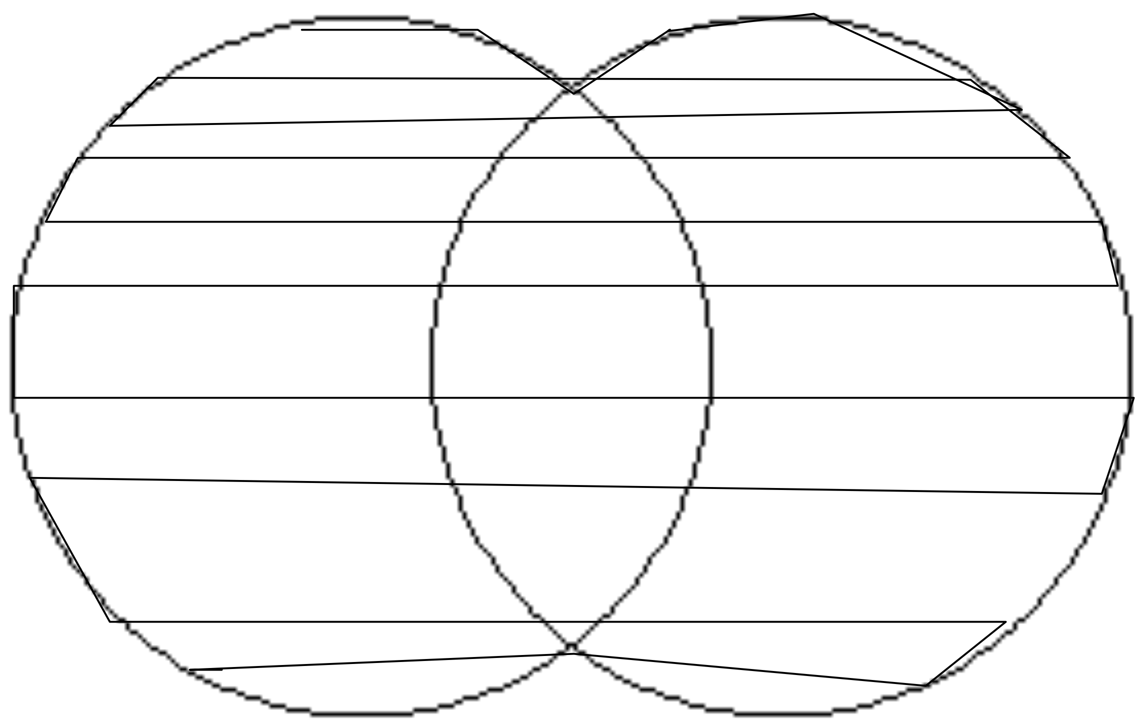
type	# favorable
LL	1
LR	1
RL	1
RR	1

For persons “aa” in the gene of handedness.

INTERSECTION



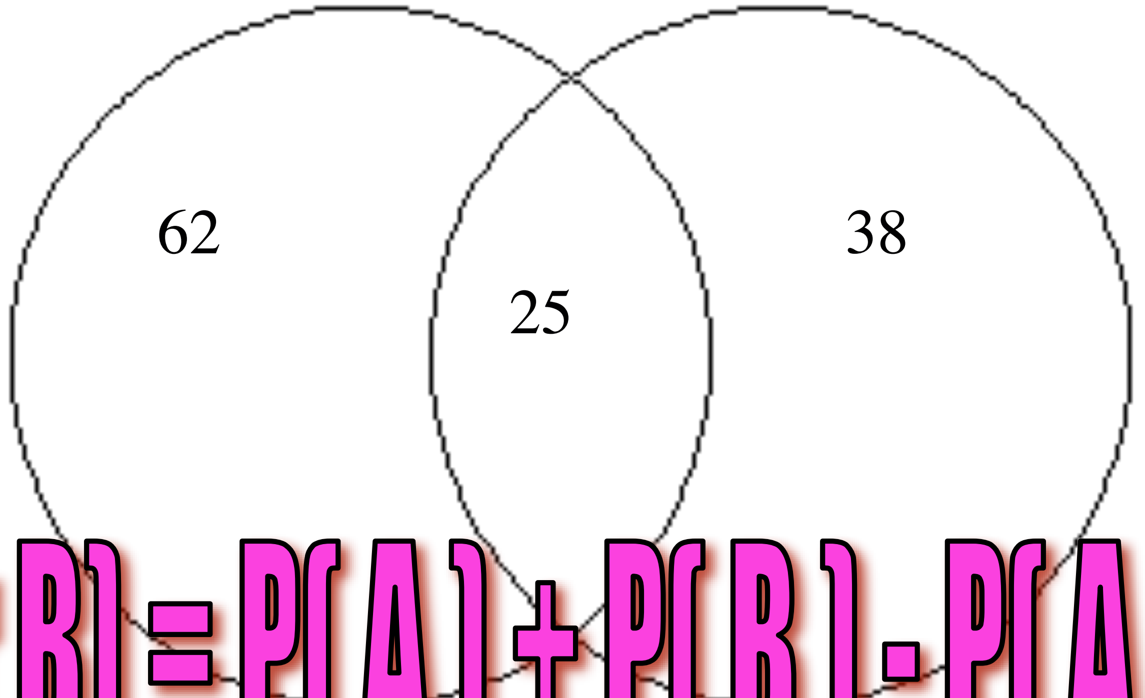
UNION



ADDITION RULE

n(union) = n(A) + n(B) - n(intersection)

62 + 25 + 38 = (62 + 25) + (25 + 38) - (25 counted twice)



union

intersection

P(A or B) = P(A) + P(B) - P(A and B)

ADDITION RULE

If there is 80% chance of rain today and 55% chance of rain tomorrow we cannot say what is the chance of rain today **or** tomorrow.

intersection

If we have also a 42% chance of rain **both** days then

union

P(rain today or tomorrow)
= .8 + .55 - .42 = .93.

ADDITION RULE WITH INDEPENDENCE

If there is 80% chance the left engine fails and 55% chance the right engine fails and if these failures are INDEPENDENT then

intersection

P(both fail) = .8 (.55) = .44

union

P(at least one fails)
= .8 + .55 - .44 = .91.